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# MAHAVIR SENIOR MODEL SCHOOL

## Half Yearly Examination

## Class:-XI

## Session 2023-24

## Mathematics

Time: 3 hours Maximum marks: 80

### General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

### SECTION-A ( 1 MARK EACH )

Q1. If X and Y are two sets and X′ denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to

(A) X (B) Y (C) φ (D) $X \cap Y$

Q2. If A and B are two sets, then $A \cap (A \cup B)$ equals

(A) A (B) B (C) φ (D) $A \cap B$

Q3. The domain of the function f defined by $f(x) = \frac{1}{\sqrt{x - |x|}}$

(A) R (B) $R^+$ (C) $R^-$ (D) None of these

Q4. Let n (A) = m, and n (B) = n. Then the total number of non-empty relations that can be defined from A to B is

(A) mn (B) $n^m - 1$ (C) $m^n - 1$ (D) $2^{mn} - 1$

Q5. If sin θ and cos θ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation.

(A) $a^2 + b^2 + 2ac = 0$ (B) $a^2 - b^2 + 2ac = 0$ (C) $a^2 + c^2 + 2ab = 0$ (D) $a^2 - b^2 - 2ac = 0$

Q6. The greatest value of sin x cos x is

(A) 1 (B) 2 (C) 2 (D) ½

Q7. Let $x, y \in R$, then x + iy is a non real complex number if:

(A) $x = 0$ (B) $y = 0$ (C) $x \neq 0$ (D) $y \neq 0$

Q8. The equation $|z+1-i| = |z-1+i|$ represents a

(A) straight line (B) circle (C) parabola (D) hyperbola

Q9. $1 + i^2 + i^4 + i^6 + \ldots + i^{2n}$ is

(A) positive (B) negative (C) 0 (D) can not be evaluated

Q10. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then

(A) breadth > 20 cm (B) length < 20 cm (C) breadth $x \geq 20$ cm (D) length $\leq 20$ cm

Q11. If $-3x + 17 < -13$, then

(A) $x \in (10, \infty)$ (B) $x \in [10, \infty)$ (C) $x \in (-\infty, 10]$ (D) $x \in [-10, 10)$

Q12. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?

(A) 72 (B) 144 (C) 14 (D) 19

Q13. In how many ways a committee consisting of 3 men and 2 women, can be chosen from 7 men and 5 women?

(A) 45 (B) 350 (C) 4200 (D) 230

Q14. All the letters of the word 'EAMCOT' are arranged in different possible ways. The number of such arrangements in which no two vowels are adjacent to each other is

(A) 360 (B) 144 (C) 72 (D) 54

Q15. The number of terms in the expansion of $(a + b + c)^n$, where $n \in N$ is

(A) $\frac{(n+1)(n+2)}{2}$ (B) $n + 1$ (C) $n + 2$ (D) $(n + 1)\, n$

Q16. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the G.P. is

(A) -4/5 (B) 1/5 (C) 4 (D) none the these

Q17. The minimum value of the expression $3^x + 3^{1-x}$, $x \in R$, is

(A) 0 (B) 1/3 (C) 3 (D) 2/3

Q18. The distance of point P(3, 4, 5) from the yz-plane is

(A) 3 units (B) 4 units (C) 5 units (D) 550

Q19. What is the length of foot of perpendicular drawn from the point P (3, 4, 5) on y-axis

(A) $\sqrt{41}$ (B) $\sqrt{34}$ (C) 5 (D) none of these

Q20. Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(A) $\sqrt{50}$ (B) 3 (C) 4 (D) 5

Q21. If A = {x : x is a natural number }, B = {x : x is an even natural number} C = {x : x is an odd natural number} and D = {x : x is a prime number },
find (i) A ∩ B (ii) A ∩ C

OR

Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

Q22. Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Q23. Show that tan3x tan2x tanx = tan3x – tan2x – tanx

Q24. Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.

Q25. Find the number of different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that (i) all vowels occur together (ii) all vowels do not occur together.

SECTION-C (3 MARKS EACH)

Q26. Let f = {(1,1), (2,3), (0,–1), (–1, –3)} be a function from Z to Z defined by f(x) = ax + b, for some integers a, b. Determine a, b.

Q27. Prove that $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

Q28. If α and β are different complex numbers with |β| =1 , then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Q29. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}) : (3-2\sqrt{2})$

OR

If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A\pm\sqrt{(A+G)(A-G)}$

Q30.Find $(a+b)^4 - (a-b)^4$ . Hence, evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$.

Q31. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2 R^n = S^n$

SECTION-D ( 5 MARKS EACH)

Q32. Prove that $\cos 2x \cos\frac{x}{2} - \cos 3x \cos\frac{9x}{2} = \sin 5x \sin\frac{5x}{2}$

Q33. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl ? (ii) at least one boy and one girl ? (iii) at least 3 girls?

OR

How many numbers greater than 1000000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Q34. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that $(q + p) : (q - p) = 17:15$

Q35. Let f(x)= $\left\{\left(x, \frac{x^2}{1+x^2}\right)\right\}$ be a function from R into R. Determine the range of f.

SECTION-E (4 MARKS EACH)

Q36. Read the following passage and answer the question given below. The longest river of North America is Mississippi River In how many ways can the letters of the word MISSISSIPPI be arranged Such that

(i) All letters are used

(ii) All I"s are together

(iii) All I"s are not together OR All S "s are not together

Q37. If $\sin x = \frac{1}{4}$ where x lies in 2nd Quadrant and $\frac{x}{2}$ is circular half of angle x.

Find a) $\sin\frac{x}{2}$

b). $\cos\frac{x}{2}$

c) $\tan\frac{x}{2}$ OR sin2x

Q38. A manufacturer has 600 litres of a 12% solution of acid.

a) How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15%?

b). How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be less than 18%?